Complementarity

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Bohr's well documented opposition to Einstein's corpuscular theory of light abated in lieu of the Bothe- Geiger experiments, in which the particle nature of radiative phenomena manifested itself. These results were in blatant contradiction with the Bohr-Kramer-Slater interpretation of the interaction between atomic systems. The B-K-S paper assumed that the radiative aspects of atomic transitions were solely describable in terms of the "wave picture". It was this descriptive contrast which prompted Bohr to find a harmonious relationship between the particle-wave nature of the radiative aspects of quantum interactions.

Bohr first discussed the wave-particle dilemma at length in a lecture delivered to the Copenhagen Academy on December 17, 1926, after which he declared to Hoffding that he had become "...more and more convinced of the need of a symbolism if one wants to express the latest results of Physics" It appears that Bohr began to rigorously develop his ideas on complementarity in early 1927. It was during this time that he associated his perception of wave particle duality with the quantum postulate, according to which there is only a finite degree of determinism at the atomic level. This was in blatant contradiction with classical reasoning, and arises from Plank's quantum of action which has to be attributed to every atomic process.

Bohr's "Principle of Complementarity" was a gradually developed set of intuitive notions , brought about by his reluctant acceptance of wave-particle duality. It was Bohr's opinion that the underlying logic of quantum theory (as opposed to its formalism) ought to be exploited in order to reach an appropriate interpretation. In the fundamental equations

$$E = h\nu; \qquad p = hk,$$

Bohr asserted that proportionality had been established between two mutually exclusive yet equally necessary descriptive aspects of nature. For here, particle attributes (E, p) necessary for describing the interaction of radiation and matter had been related to factors (ν, k) necessary for the description of the propagation of light. He thereby denoted a new logical instrument; *Complementarity*—the relation between complimentary yet mutually exclusive descriptions or sets of concepts which are both necessary for an exhaustive description of events.

In Heisenberg's uncertainty relations he saw, rather than a piece of mathematical formalism a statement concerning the degree to which complementary notions may overlap. Even though the two descriptive concepts were mutually exclusive, he saw also that a logical contradiction could never be reached, because the uncertainty relations show that the sharp exhibition of one possible characteristic is to the detriment of the other. Thus different experimental configurations are required for observable manifestations of complementary aspects of the phenomenon (as when one characteristic manifests itself in the object/measurement system it is to the exclusion of the other).

So fundamentally the uncertainty relations declare that "....no physical situation can arise which exhibits simultaneously and rigorously (sharply) both complementary aspects of the phenomenon." Bohr felt that using mutually contradictory notions for the description of the same physical situation was justifiable owing to the indeterminateness of the term "observation". Because of the quantum of action associated an impossibility arises when one tries to distinguish between the object and the measuring device (whose behaviour can be described in classical terms). Since the object perturbs the device in some manner to produce data, the object's complimentary feature (not under consideration) is altered in some way. By combining the atomic system with different classically describable devices, Bohr declared that one may measure complimentary variables. By expressing these further results classically, a descriptive picture of the system may be constructed in terms of complimentary classical features.

Bohr first presented his views on complementarity to his peers on September 16, 1927 in Como. He delivered a lecture on "The quantum postulate and the recent development of atomic theory". He began (I assume without pausing for breath) "I shall try to by making use only of simple considerations and without going into any details of technical mathematical character to describe to you a certain general point of view which I believe is suited to giving an impression of the general trend of the development of the theory from its very beginning and which I hope will be helpful in order to harmonise the apparently conflicting views held by different scientists" He then stressed the difference between the classical description, which assumes observation may take place without appreciable disturbance and the quantum postulate which assumes that an essential discontinuity has to be associated with quantum measurement.

Bohr then continued "On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, any observation will be impossible, and above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible and there can be no question of causality in the ordinary sense of the word. The very nature of the quantum theory thus forces us to regard the space time co-ordinates and the claim of casuality, the union of which characterizes the classical theories, as complimentary but exclusive features of the description, symbolizing the idealisation of observation and definition respectively." This last statement encapsulated what has become known as the "Copenhagen Interpretation" of quantum mechanics.

His concluding remarks during a discussion with Born, Kramer, Heisenberg et al, were to the effect that the new formalism of quantum mechanics was a symbolic scheme which permits only predictions of results obtainable under conditions specified in terms of classical concepts. Arguably one of the greatest minds of all time, Albert Einstein, maintained throughout his illustrious career a deterministic approach to atomic measurement. He and Neils Bohr were famous antagonists, at least as far as the validity of quantum mechanics was concerned. Einstein held to the classical opinion that a division could be made between the agency of measurement and the object and furthermore a discrete value could be associated with a property of the object. This assumption may extended further to that made by classical physics, which declares that the property which is being measured objectively exists before the interaction between instrument of measurement and object. This is in direct contradiction with quantum theory which is best illustrated by example. Consider the historic Stern-Gerlach experiment of 1922, whose purpose was to measure the possible values of the magnetic dipole moment for silver atoms, by measuring the deflection of a neutral atomic beam by an inhomogeneous magnetic field.

The Hamiltonian for the atomic system is

$$H = \frac{\vec{p}^2}{2m} - \vec{\mu} \cdot \vec{B} \tag{1}$$

where m is the mass of the atom, $\vec{p}$ it's momentum, $\vec{\mu}$ it's intrinsic angular moment and $\vec{B}$ the applied magnetic field. Mu may be due to some internal rotation about a symmetry axis, thus let

$$\vec{\mu} = g\vec{S}$$

where $\vec{S}$ is the angular momentum around the centre of mass of the atom and g depends on the mass and charge distribution about this rotation axis. Also,

$$\vec{B} = \vec{B}(\vec{r})$$

where $\vec{r}$ is the position vector for the centre of mass. The equations of motion may be obtained from the Poisson brackets;

$$\dot{\vec{r}} = \{\vec{r}, H\} = \frac{\vec{p}}{m} \tag{2}$$

$$\dot{\vec{p}} = \{\vec{p}, H\} = \nabla(\vec{\mu} \cdot \vec{B}) \tag{3}$$

$$\dot{\vec{\mu}} = \{\vec{\mu}, H\} = g(\vec{\mu} \times \vec{B}) \tag{4}$$

Eq. (4) suggests that $\vec{\mu}$ precesses around the direction of $\vec{B}$, which cannot be constant since this would violate the Maxwellian equation $\nabla \cdot B = 0$. We may solve approximately if $\overline{\vec{B}}$ is greater than the variation of $\vec{B}$ in the magnet gap. Also the atom's time in the field must be much larger than its precession time $(2\pi/gB)$. Subject to these conditions the atom will precess many times around the direction of $\vec{B}$, so that μ_x, μ_y may be neglected on time average. Then let $\overline{\vec{B}} = B\mathbf{e}_1$ where $\mathbf{e}_1$ is the unit vector in the direction of orientation of the magnet (the *measuring device*) and B is the magnitude of $\vec{B}$.

From Eq. (4), it follows that $\vec{\mu}\mathbf{e}_1$ is a constant and on time average may be replaced by $\mu_1\mathbf{e}_1$, where $\mu_1 = \vec{\mu}\mathbf{e}_1$.

Then from Eq. 3, we have

$$\frac{d}{dt}(\mathbf{e}_1 \cdot \vec{p}) = \mu_1 B'$$

where $B' = (\vec{e}_1 \cdot \nabla)(\vec{e}_1 \cdot \vec{B})$ depends on the construction of the magnet, i.e., the measuring device. $\dot{\vec{p}}$ acts during a time L/v, where v is the longitudinal velocity of the atoms and L is the length of the magnet. The transverse momentum imparted to the atoms by this force is $\mu_1 B' L/v$ and their deflection angle is $\mu_1 B' L/2E$ where $E = \frac{1}{2}mv^2$. All terms except for μ_1 are determined by the experimental set up and are classically defined parameters, which are fixed for a given experiment.

By the intensity distribution formed on the glass plate some eight hours later, Stern and Gerlach determined that μ_1 could only take on one of only two values, $\pm|\vec{\mu}|$. Classically, μ_1 should be able to take on any value between and including the aforementioned. A fundamental contradiction arises. The orientation of the magnet could be changed as thus making angles of $\pm 120^0$ with $\mathbf{e}_1$, thus

$$\mu_2 = \vec{\mu} \cdot \mathbf{e}_2$$

or

$$\mu_3 = \vec{\mu} \cdot \mathbf{e}_3$$

Since physical laws are not dependent on measuring devices, Stern and Gerlach would have found similarly that

$$\mu_2 = \pm|\vec{\mu}|$$

or

$$\mu_3 = \pm|\vec{\mu}|$$

; but

$$\mu_1 + \mu_2 + \mu_3 = \vec{\mu}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3) = 0$$

. Now μ_1, μ_2, and μ_3 cannot simultaneously be $\pm|\vec{\mu}|$ and sum to zero-thus there arises an inseparability between the measuring device (the magnet; its orientation) and the quantized nature of the magnetic dipole moment. Measurement may not be described as "..the acquisition of knowledge about some objective pre-existing reality" since a contradiction is reached, In the S-G experiment discrete values were associated with a vector that can be continuously rotated thus "the meaning of these discrete values cannot be that of objective vector components, which would be independent of the measuring process."

The quantum interpretation of measurement was eloquently put by E. C. Kemble;

> We have no satisfactory reason for ascribing objective existence to physical quantities as distinguished from the numbers obtained when we make the measurements which we correlate with them. There is no real reason for supposing that a particle has at every moment a definite, but unknown, position which may be revealed by a measurement of the right kind, or a definite momentum which can be revealed by a different measurement. On the contrary, we get into a maze of contradiction as soon as we inject into quantum mechanics such concepts as carried over form the language and philosophy of our ancestors.. It would be more exact if we spoke of 'making measurements' of this, that, or the other type instead of saying that we measure this, that, or the other 'physical quantity'.

Born and Heisenberg's address at Como suggested that the determinism which had been accepted as the basis of the exact sciences to date must be abandoned. Furthermore, they declared that "Every additional advance in our understanding of the formulas has shown that a consistent interpretation of the quantum mechanical formalism is possible only on the assumption of a fundamental indeterminism...." In 1949, on the occasion of Einstein's 70th birthday, a thick volume was released dedicated to the founder of relativity theory. It contained articles contributed by scholars from eleven countries, with contributors such as Born, Pauli et al, who largely expressed their disdain concerning Einstein's negativism toward quantum theory. Without doubt, the most absorbing of studies was that by Bohr, who recounted the development of quantum theory and debates with Einstein. At the conclusion, Einstein faces his accusers, boldly declaring his inability to accept that the wave function ψ can give the complete description for a the state of an atomic system. He believed that the wave function provided a description for a collage of identical systems rather than for one in particular. His argument, as always, was that it is possible to obtain an image of reality independent of the measurement process (the observer). If one admitted its existence, then the prevalent viewpoint belonging to the current interpretation of quantum theory must be discarded. The fact that two of the greatest minds of our time could hold opposing views on the quantum theory points to the complexity of its interpretational nature.

It is only fair however that the founder of relativity be given the dignity of the concluding remark: "I look upon quantum mechanics with admiration and suspicion”